

Entropic destruction of a rotating heavy quarkonium

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Using the AdS/CFT duality, we study the destruction of a rotating heavy quarkonium due to the entropic force in $\mathcal{N} = 4$ SYM theory and a confining YM theory. It is shown that in both theories increasing the angular velocity leads to decreasing the entropic force. This result implies that the rotating quarkonium dissociates harder than the static case.

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I. INTRODUCTION

The experiments of ultrarelativistic nucleus-nucleus collisions at RHIC and LHC have produced a new state of matter so-called quark gluon plasma(QGP) [1–3]. One important signal of the formation of QGP is quarkonium suppression [4]. However, the recent experimental research of quarkonium production in nuclear collisions has shown a puzzle: the charmonium suppression at RHIC (lower energy density) is stronger than that at LHC (larger energy density) [5, 6]. This is obviously in contradiction with the Debye screening assumption [4] and the thermal activation scenario [7].

To explain this contradiction, some authors suggest that the recombination of the produced charm quarks into charmonia may be a solution [8, 9]. But recently it was argued [10] that this puzzle is related to the nature of deconfinement, based on the Lattice results [11–13] which indicate that a large amount of entropy associated with the heavy quark-antiquark pair placed in the QGP. It was originally argued in [10] that the entropic force is responsible for dissociating the quarkonium and this force can be related to the entropy S , that is

$$F = T \frac{\partial S}{\partial L}, \quad (1)$$

where T is the temperature of the plasma.

AdS/CFT [14–16], which relates a d -dimensional quantum field theory with its dual gravitational theory, living in $(d+1)$ dimensions, has yielded many important insights into the dynamics of strongly-coupled gauge theories. In this approach, K. Hashimoto et al have first analyzed the entropic force associated with the heavy quark pair [17], based on the calculations of the quark-antiquark potential from AdS/CFT [18–20]. It is found [17] that the peak of the entropy near the transition point is related to the nature of deconfinement. Sooner after [17] studies of the entropic destruction of a moving heavy quarkonium has been discussed in [21], the authors showed that by increasing the velocity the moving quarkonium dissociates easier than the static ones.

In this paper, we extend the holographic studies of [17] by setting the quarkonium to have a angular velocity. We would like to see how the angular velocity affects the entropic force or the quarkonium dissociation. It is the motivation of the present work.

The paper is organized as follows. In the next section, we investigate the entropic force of a rotating quarkonium in $\mathcal{N} = 4$ SYM theory. In section 3, the entropic force of a rotating quarkonium is studied in a confining YM theory as well. The last part is devoted to conclusion and discussion.

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II. ENTROPIC FORCE OF A ROTATING QUARKONIUM IN $\mathcal{N} = 4$ SYM THEORY

We now analyze the behavior the entropic force associated with a rotating heavy quark pair in $\mathcal{N} = 4$ SYM theory. The metric is given by

$$ds^2 = -\frac{r^2}{R^2}f(r)dt^2 + \frac{r^2}{R^2}d\vec{x}^2 + \frac{R^2}{r^2}\frac{1}{f(r)}dr^2 + R^2d\Omega_5^2, \quad (2)$$

where $f(r) = 1 - \frac{r_h^4}{r^4}$, r denotes the radial coordinate describing the 5th dimension. R is the AdS radius. The event horizon is located at $r = r_h$ with $r_h = \pi R^2 T$, where T is the temperature of the black hole. $d\Omega_5$ is the element of the solid angle of S^5 . To consider the rotation, one can introduce a angular momentum in some ϕ -direction [22–25]. For simplicity, we here consider one rotational motion direction, i.e. $d\Omega_5 = d\phi$.

To proceed, we follow the calculations of [17] with the metric of (2). The Nambu-Goto action is

$$S = T_F \int d\tau d\sigma \mathcal{L} = T_F \int d\tau d\sigma \sqrt{g}, \quad (3)$$

where $T_F = \frac{1}{2\pi\alpha'}$ is the fundamental string tension. $\frac{R^2}{\alpha'} = \sqrt{\lambda}$ with λ the 't Hooft coupling. g stands for the determinant of the induced metric

$$g_{\alpha\beta} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}, \quad (4)$$

where X^μ and $g_{\mu\nu}$ are the target space coordinates and the metric respectively.

For our assumption, we choose the static gauge,

$$x^0 = \tau, \quad x^1 = \sigma, \quad (5)$$

and assume that the coordinate r depends on σ and the angular direction ϕ depends on τ

$$r = r(\sigma), \quad \phi = \phi(\tau), \quad (6)$$

then the induced metric is found to be

$$g_{00} = \frac{r^2}{R^2} \left(1 - \frac{r_h^4}{r^4}\right) + R^2(\phi')^2, \quad g_{01} = g_{10} = 0, \quad g_{11} = \frac{r^2}{R^2} + \frac{R^2}{r^2} \left(1 - \frac{r_h^4}{r^4}\right)^{-1} (\dot{r})^2, \quad (7)$$

with $\phi' = \frac{\partial \phi}{\partial \tau}$ and $\dot{r} = \frac{\partial r}{\partial \sigma}$.

The Lagrangian density \mathcal{L} becomes

$$\mathcal{L} = \sqrt{(\dot{r})^2 + \frac{r^4}{R^4} \left(1 - \frac{r_h^4}{r^4}\right) + \frac{R^4}{r^2} \left(1 - \frac{r_h^4}{r^4}\right)^{-1} (\dot{r})^2 (\phi')^2 + r^2 (\phi')^2}. \quad (8)$$

Notice that \mathcal{L} dose not depend on σ explicitly, so the Hamiltonian density is constant, that is

$$H = \mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{r}} \dot{r} = \text{constant}. \quad (9)$$

This constant can be found at the special point $r(0) = r_c$ with $\dot{r} = 0$, as

$$H = \sqrt{\frac{r_c^4}{R^4} \left(1 - \frac{r_h^4}{r_c^4}\right) + r_c^2 (\phi')^2}. \quad (10)$$

Following (8), (9) and (10), one has a differential equation

$$\dot{r} = \frac{dr}{d\sigma} = \sqrt{\frac{a^2(r) - a(r)a(r_c)}{a(r_c)b(r)}}, \quad (11)$$

where

$$a(r) = \left(\frac{r}{R}\right)^4 f(r) + r^2 (\phi')^2, \quad a(r_c) = \left(\frac{r_c}{R}\right)^4 f(r_c) + r_c^2 (\phi')^2, \quad b(r) = 1 + \frac{R^4}{r^2 f(r)} (\phi')^2, \quad (12)$$

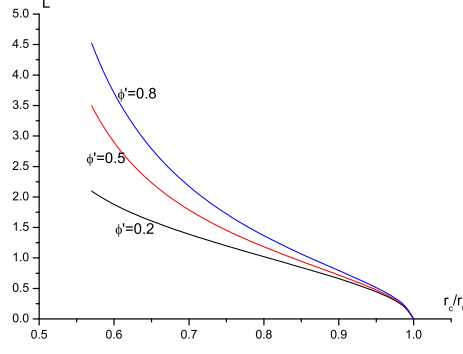


FIG. 1: The inter-distance L versus r_c/r_0 at a fixed temperature $r_t/r_0 = 0.5$. Here we take $R = 1$.

ϕ'	0	0.01	0.03	0.05	0.08
c	0.240	0.246	0.249	0.253	0.270

TABLE I: The values of c for some angular velocity in $\mathcal{N} = 4$ SYM theory.

with

$$f(r_c) = 1 - \left(\frac{r_h}{r_c}\right)^4. \quad (13)$$

By integrating (11) the inter-quark distance L can be calculated as

$$L = 2 \int_{r_c}^{r_0} dr \sqrt{\frac{a(r_c)b(r)}{a^2(r) - a(r)a(r_c)}}. \quad (14)$$

where $r_0 = \infty$ is the boundary.

On the other hand, the on-shell action of the fundamental string in the dual theory is related to the free energy of the quark anti-quark pair. For small inter-quark distance L , the fundamental string is connected and its on-shell action can be expressed as

$$F^{(1)} = 2T_F \int_{r_h}^{r_0} dr \sqrt{\frac{a(r)b(r)}{a(r) - a(r_c)}}. \quad (15)$$

If the distance L is large enough, the fundamental string will break in two pieces implying the quarks are screened. For this case, the free energy is $F^{(2)}$. However, the choice of $F^{(2)}$ is not unique [26]. We here choose a configuration of two disconnected trailing drag strings [27], that is

$$F^{(2)} = 2T_F \int_{r_h}^{r_0} dr. \quad (16)$$

To proceed further, we have to resort to numerical methods. In Fig 1, we plot the inter-quark distance L as a function of r_c/r_0 at a fixed temperature $r_t/r_0 = 0.5$ for three different angular velocity. In the plots from top to bottom $\phi' = 0.8, 0.5, 0.2$ respectively. One can see clearly that as ϕ' increases the inter-quark distance increases. Or in other words, the faster the angular velocity, the farther the distance of the heavy quark pair. This can be also understood by considering the centrifugal force which may have the effect of increasing L .

In addition, the numerical results show that there exist a const c which is dependent of ϕ' . If $L > \frac{c}{T}$ the quarks are completely screened. Here we present the values of c for some different ϕ' in table 1. We can see that increasing ϕ' leads to increasing c . However, we do not find the values of c for large ϕ' . This is curious.

Next, we calculate the entropy as $S = -\frac{\partial F}{\partial T}$. For the screened case $L > \frac{c}{T}$, one finds

$$S^{(2)} = \sqrt{\lambda} \theta \left(L - \frac{c}{T} \right), \quad (17)$$

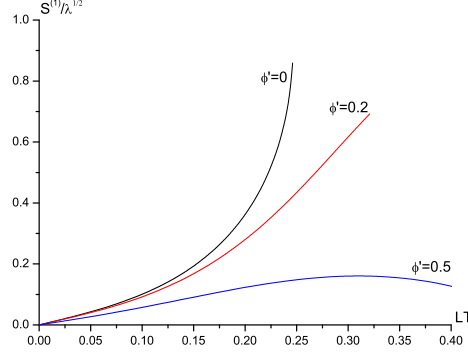


FIG. 2: The entropy $S^{(1)}/\sqrt{\lambda}$ against LT in $\mathcal{N} = 4$ SYM theory. Here we take $R = 1$.

which implies the entropy at large distance is constant and independent of the temperature.

For $L < \frac{c}{T}$, to evaluate the entropic force, we study the growth of the entropy $S^{(1)}$ against the inter-quark distance. In Fig.2, we set $R = 1$ and plot $S^{(1)}/\sqrt{\lambda}$ as a function of LT with three different ϕ' . We can see that at small distances by increasing the angular velocity the entropy decreases. Interestingly, if the angular velocity is large enough, the entropy even decreases as LT increases at large LT .

As stated above, the entropic force, related to the growth of the entropy with the distance, is responsible for the destruction of the quarkonium. From the figures, one finds that increasing the angular velocity leads to decreasing the entropic force. Also, if the angular velocity is large enough, the entropic force even becomes "negative" at large LT . As a result, we conclude that in the $\mathcal{N} = 4$ SYM theory, the entropic force destructs the rotating quarkonium harder than the static case.

III. ENTROPIC FORCE OF A ROTATING QUARKONIUM IN A CONFINING YM THEORY

Next, we investigate the entropic force in a confining YM theory. To analyze the entropic force around the deconfinement transition, one should opt for a theory which is confined at low energy and deconfined at high temperature. The confining $SU(N)$ gauge theory based on ND4 brans on a circle [29] satisfies these conditions. In a deconfined phase, the metric of this theory is [17]

$$ds^2 = \left(\frac{r}{R}\right)^{3/2}[-f(r)dt^2 + (d\vec{x})^2 + (dx_4)^2] + \left(\frac{R}{r}\right)^{3/2}\left[\frac{1}{f(r)}dr^2 + r^2 d\Omega_4^2\right], \quad (18)$$

with

$$f(r) = 1 - \left(\frac{r_h}{r}\right)^3, \quad (19)$$

where the horizon is fixed at $r = r_h$. The temperature of this geometry is given by

$$T = \frac{3}{4\pi} \frac{\sqrt{r_t}}{R^{3/2}}. \quad (20)$$

Parallel to the $\mathcal{N} = 4$ SYM case in the previous section, we also choose one rotational motion direction as $d\Omega_4 = d\phi$. The Lagrangian density \mathcal{L} reads

$$\mathcal{L} = \sqrt{\left(\frac{r}{R}\right)^3 f(r) + r^2 (\phi')^2 + (\dot{r})^2} + \frac{R^3}{r f(r)} (\phi')^2 (\dot{r})^2. \quad (21)$$

with $\phi' = \frac{\partial \Phi}{\partial \tau}$ and $\dot{r} = \frac{\partial r}{\partial \sigma}$, where we have used the condition (5) and (6).

We again have a conserved quantity as

$$H = \mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{r}} \dot{r} = \text{constant}. \quad (22)$$

ϕ'	0	0.05	0.1	0.15	0.2
c	0.252	0.253	0.256	0.261	0.270

TABLE II: The values of c for some angular velocity in a confining YM theory.

By using the boundary condition at $\sigma = 0$,

$$\frac{dr}{d\sigma} = 0, \quad r = r_c, \quad (23)$$

we have a differential equation

$$\dot{r} = \frac{dr}{d\sigma} = \sqrt{\frac{a^2(r) - a(r)a(r_c)}{a(r_c)b(r)}}. \quad (24)$$

where

$$a(r) = \left(\frac{r}{R}\right)^3 f(r) + r^2(\phi')^2, \quad a(r_c) = \left(\frac{r_c}{R}\right)^3 f(r_c) + r_c^2(\phi')^2, \quad b(r) = 1 + \frac{R^3}{rf(r)}(\phi')^2, \quad (25)$$

with

$$f(r_c) = 1 - \left(\frac{r_h}{r_c}\right)^3. \quad (26)$$

By solving (24), one finds the inter-distance of the $Q\bar{Q}$ as

$$L = 2 \int_{r_c}^{r_0} dr \sqrt{\frac{a(r_c)b(r)}{a^2(r) - a(r)a(r_c)}}, \quad (27)$$

with $r_0 = \infty$ the boundary.

Now we proceed to evaluate $F^{(1)}$ and $F^{(2)}$ with numerical methods, we find that if $L > \frac{c}{T}$ the quarks are screened. The values of c with respect to ϕ' are shown in Table 2.

In the case of $L > \frac{c}{T}$, we find

$$F^{(2)} = 2T_F \int_{r_h}^{r_0} dr, \quad S^{(2)} = \sqrt{\lambda}\theta(L - \frac{c}{T}). \quad (28)$$

For $L < \frac{c}{T}$, we have

$$F^{(1)} = 2T_F \int_{r_h}^{r_0} dr \sqrt{\frac{a(r)b(r)}{a(r) - a(r_c)}}. \quad (29)$$

After calculating the entropy $S^{(1)}$ as $S^{(1)} = -\frac{\partial F^{(1)}}{\partial T}$, we plot $S^{(1)}/\sqrt{\lambda}$ against LT with three different ϕ' in Fig.3, one can see that the behavior of the entropy against LT in a confining YM theory is very similar to the case of $\mathcal{N} = 4$ SYM theory, the only difference is the slope of the curves. In this case, we also find that increasing the angular velocity leads to decreasing the entropic force at small distances. Thus, one concludes that in a confining YM theory, the entropic force destructs the rotating quarkonium harder than the static case as well.

IV. CONCLUSION AND DISCUSSION

In this paper, we have investigated the destruction of a rotating heavy quarkonium due to the entropic force from the AdS/CFT. The effect of a nonzero angular velocity on the entropic force in $\mathcal{N} = 4$ SYM theory and a confining YM theory has been studied. It is shown that in both theories, the presence of the angular velocity tends to decrease the entropic force thus making the rotating quarkonium dissociates harder than the static case. To our knowledge, this result is new and different from the previous studies, see for example in [30].

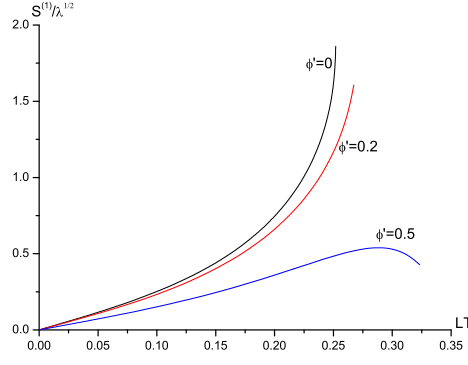


FIG. 3: The entropy $S^{(1)}/\sqrt{\lambda}$ against LT in a confining YM theory. Here we take $R = 1$.

Interestingly, it was argue [25] that increasing the inter-distance of $Q\bar{Q}$ can be regarded as decreasing the horizon r_h or decreasing the gravity effect. Since r_h is a increasing function of T , increasing of the angular velocity leads to decreasing of the system temperature. As we know, lower system temperature makes the $Q\bar{Q}$ harder to dissociate. Thus, this agreement also supports that the rotating quarkonium dissociates harder than the static ones.

Finally, the effect of the rotating heavy quarkonium may be an explanation to the puzzle on the suppression of the charmonium at RHIC and LHC: the higher the energy density, the stronger rotating of the quarkonium, one step further, the smaller the entropic force, the harder the heavy quarkonium dissociates.

V. ACKNOWLEDGMENTS

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